Investigating Slingshot Trajectory and Pull Distance

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ABSTRACT

The distance of launch for a small soft projectile from a slingshot was studied. The distance to pull the slingshot back and the angle from the ground to launch the projectile from were considered. The optimal combination of these two was desired to maximize the launch distance of the projectile. A second order model was obtained to predict launch distance as a function of pull distance and launch angle. Both were found to have a positive effect on launch distance. Optimal settings for pull distance were found to be in the range of 4 to 6 inches, and optimal settings for launch angle were found to be in the range of 40 to 50 degrees.

I. INTRODUCTION

The laws of physics tell us that, when launching a projectile through the air, there is a horizontal and vertical component to its velocity. The ways in which these interact determines how far the projectile can fly. The x and y components are determined by, among other factors, the angle at which you launch the projectile, and how much force the projectile has behind it when it launches. This begs the question, what is the best combination of these to launch the projectile the furthest? This information is vital to know when you are trying to use a slingshot to launch a soft projectile, a scenario which many people find themselves in at social gatherings, like birthday parties, with some form of friendly competition involved. Winning friendly competitions is a great opportunity to gain social status with the people present, so understanding the best way to use a slingshot is important. The purpose of this research is to determine the best combination of launch angle and distance that you stretch a slingshot to maximize the distance that a soft projectile is launched. We accomplished this using a factorial design of experiments and response surface methodology, to allow for more precision in determining the optimal settings of our factors of interest.

II. MATERIALS AND METHODS

An experiment was carried out to investigate how the launch angle from the ground and the pull distance of the sling shot affects the travel distance of a payload (one small 5-gram foam ball). Levels to these experimental factors can be found in Table 1.

Table 1
Factor Levels

Launch Angle (°)	Pull Distance (Inches)	Coded Launch Angle	Coded Pull Distance
25	2.5	-1	-1
65	2.5	1	-1
25	7.5	-1	1
65	7.5	1	1

The travel distance was measured in inches by a measuring tape. The small 5-gram foam ball was held constant as the payload for all runs. Each experimental run consisted of one launch of the payload. The design was a Uniformly Precise, Rotatable Central Composite design with 4 axial runs, 4 factorial runs, 5 center runs (n_{total} = 13).

Table 2
Experimental matrix for the central composite design

Launch Angle (°)	Pull Distance (Inches)	X_A	X_P	Travel Distance (inches)
75	5	1.41	0	67.4
25	2.5	-1	-1	39.3
45	5	0	0	125.6
65	7.5	1	1	78.5
15	5	-1.41	0	84.2
45	5	0	0	138.8
45	5	0	0	138.3
45	8.5	0	1.41	133.4
45	5	0	0	132.7
65	2.5	1	-1	85.5
45	1.5	0	-1.41	53.5
45	5	0	0	111.4
25	7.5	-1	1	73.3

Experimental Protocol:
Each launch of payload, using the slingshot, was carried out on a level ground (grass field). The level ground acted as a control for distance travelled after landing. The day we conducted our experiment it was slightly windy, which might have accounted for extraneous variations in our results. Launch Angle was measured using a protractor level to the ground, whereas pull distance was measured using a white line as a point from which we began the pull away from body of the slingshot for each respective distance. The experimental runs can be found in Table 2.

Description of the Statistical Analysis:
We carried out a statistical analysis on the experimental results after experimental runs were conducted via JMP software. In order to find the best fit model, we conducted a Lack of Fit test on the first-order regression with interaction. Then, we conducted multiple Partial F-Tests through JMP's Fit Model interface. After fitting for the second-order model, normality was improved as can be seen through the less curvature in Figure 2 (second-order model Normality plot) compared to Figure 1 (first-order model Normality plot).

III. RESULTS AND DISCUSSION
We began model fitting with a first-order regression with interaction through JMP's Fit Model interface. This model included the main effects Angle and Pull Distance, along with the interaction Angle*Pull Distance, with Launch Distance as the response, so that the following expression was obtained:

$$\widehat{Launch\ Distance} = 97.07 + 3.47X_A + 17.51X_P - 10.25X_{AP}$$

However, at a 5% significance level, we found there to be statistically significant curvature (F = 16.86, df = 9, p = 0.0086), meaning this first-order model did not adequately represent the data.

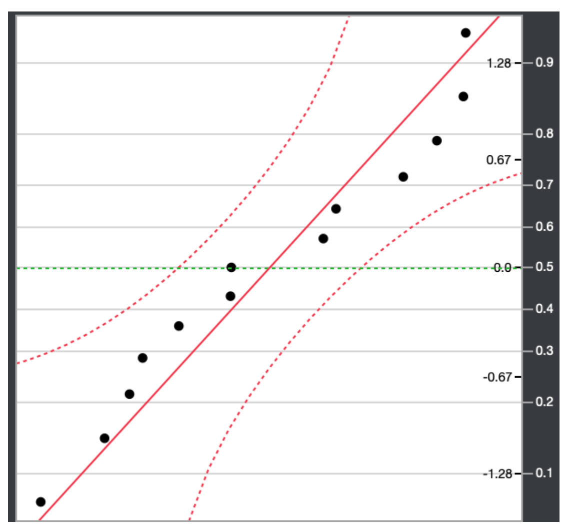

Figure 1
Normal Quantile Plot for First-Order Regression Model Residuals

Additionally, though the data did not appear significantly non-normal, there did seem to be room for improvement.
Thus, we fit a new second-order (quadratic) regression model including the main effects and interaction once again, this time adding quadratic terms Angle^2 and Pull Distance^2, giving the following expression:

$$\widehat{Launch\ Distance} = 129.4 + 3.47X_A + 17.51X_P - 30.78X_A^2 - 10.25X_{AP} - 21.91X_P^2$$

Now, at a 5% significance level, we did not find there to be a lack of fit (F = 5.40, df = 7, p = 0.068), so the second-order regression was the best possible model for this experiment. Each increase of 20° from the average launch angle causes an increase of 3.47 inches in mean travel distance, while each increase of 2.5 inches in pull distance causes an increase of 17.51 inches in mean travel distance. At the same time, since the quadratic terms of the model were significant, increasing the launch angle decreases the effect of pull distance on mean travel distance. Since our quadratic terms are both negative, the response surface for our model curves downwards in two dimensions; this means to maximize travel distance, factors will need to be near their center value (45°, 5 inches).

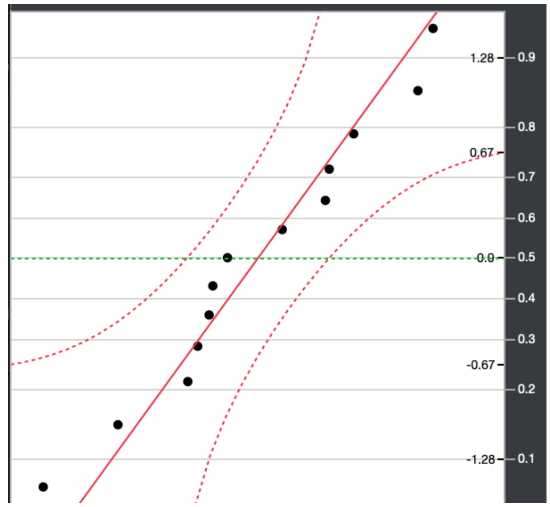

Figure 2
Normal Quantile Plot for Second-Order Regression Model Residuals

With the new model fit, Figure 2 shows its residuals tend to adhere more to the normal distribution. The second-order regression model also allowed for the use of JMP's "Maximize Desirability" function within the Prediction Profiler interface; with this, we found that launching at an angle of 44.69° from the ground with a pull distance of 5.96 inches maximizes the travel distance for our ball from the slingshot, resulting in a predicted travel distance of 132.36 inches, as can be seen in Figure 3.

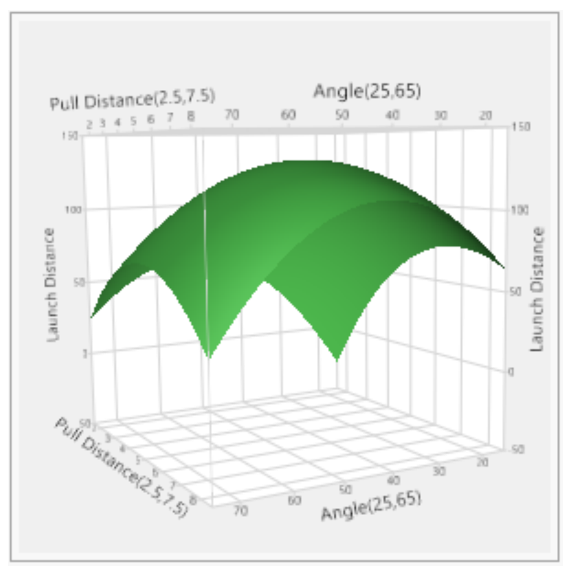

Figure 3
Launch Distance Surface Profiler

Increasing the launch angle results in an increase in the predicted launch distance when the angle is between approximately 20° and 60° from the ground, then decreases the predicted launch distance beyond that range. Meanwhile, increasing the pull distance for the sling results in an increase in the predicted launch distance when the pull distance is between approximately 2 inches and 6 inches, then decreases the predicted launch distance beyond that range.

IV. CONCLUSION

An experiment was conducted to investigate the mean travel distances based on angle and pull distance. We were able to find optimal (maximum) travel distance when the angle was 45° and pull distance was 5 inches. Similarly, we found that the Pull Distance, $Angle^2$, and Pull $Distance^2$ effects were statistically significant at the 5% level. So, it stands that pull distance and angle have an effect on the true mean launch distance of a small projectile.

V. Appendix

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	5	11515.702	2303.14	5.6377
Error	7	2859.666	408.52	Prob > F
C. Total	12	14375.368		0.0212*

Figure 4
Analysis of Variance Results

Lack Of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	3	2342.7742	780.925	6.0432
Pure Error	4	516.8920	129.223	Prob > F
Total Error	7	2859.6662		0.0574
				Max RSq
				0.9640

Figure 5
Lack of Fit Test Results

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	129.0178	9.035188	14.28	<.0001*
Angle(25,65)	3.0823529	6.932649	0.44	0.6700
Pull Distance(2.5,7.5)	17.532828	7.182011	2.44	0.0447*
Angle*Angle	-27.37856	6.988447	-3.92	0.0058*
Angle*Pull Distance	-10.25	10.10599	-1.01	0.3442
Pull Distance*Pull Distance	-23.05727	7.810903	-2.95	0.0213*

Figure 6
Parameter Estimates

Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Angle(25,65)	1	1	80.7576	0.1977	0.6700
Pull Distance(2.5,7.5)	1	1	2434.6085	5.9595	0.0447*
Angle*Angle	1	1	6270.1392	15.3483	0.0058*
Angle*Pull Distance	1	1	420.2500	1.0287	0.3442
Pull Distance*Pull Distance	1	1	3559.8415	8.7139	0.0213*

Figure 7
Effect Tests

Response Surface

Coef

	Angle(25,65)	Pull Distance(2.5,7.5)	Launch Distance
Angle(25,65)	-27.37856	-10.25	3.0823529
Pull Distance(2.5,7.5)	.	-23.05727	17.532828

Solution

Variable	Critical Value
Angle(25,65)	44.689507
Pull Distance(2.5,7.5)	5.9591311

Solution is a Maximum
Predicted Value at Solution 132.35713

Figure 8
Response Surface Profiler

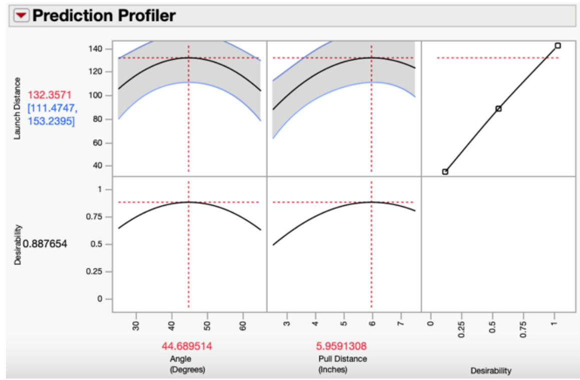

Figure 9
Prediction Profiler with Maximum Desirability